WILSON DAILY PREP
PRACTICE IS KEY

SAT MATH TOPIC SKILLS

Angles:

Angle Definitions:

- **Supplementary Angles:** two angles with a sum of 180 degrees. They usually lie on a straight line but doesn't always needs to be the case.
- **Complementary Angles:** two angles with a sum of 90 degrees. They usually form a right angle, but it doesn't always need to be the case.
- **Vertical Angles:** each of the pairs of opposite angles made by two intersecting lines.
- **Bisector** of an angle: a ray whose end point is the vertex of the angle and which divides the angle into two equal angles.

Transversals: When parallel lines get crossed by another line (called a transversal), some angle relationships are formed:

- **Corresponding angles:** All angles that have the same position with regards to the parallel lines and the transversal are corresponding pairs. They are congruent!
- **Alternate interior angles:** angles on opposite sides of the transversal and on the interior of the parallel lines. These angels are congruent! (You can trace a "Z" around these angles).
- **Alternate exterior angles:** angles on opposite sides of the transversal and on the exterior of the parallel lines. There angles are also congruent!
- **Consecutive angles** may be defined as two interior/exterior angles lying on the same side of the transversal cutting across 2 parallel lines.

Note: This holds true for any type of diagram, where there are parallel lines. For example, sometimes the parallel lines may be the two sides of a rhombus, and the "Transversal" may be the diagonal!

Angles in a Polygon: The sum of the degrees of the angles in a polygon can be found:

- using the formula $180(n - 2,)$ where n = the number of sides the given polygon
- by breaking the polygon into shapes, you know (ex: triangles or quadrilaterals)

Remember --> when you are given a question asking about an angle in a polygon, remember first you must find the total degrees in that given polygon, then you must split the total degree measurement evenly amongst the # of angels in the polygon.

Angles - Reminders

- Vertical angles = opposite angles = congruent
- Angles that lie on a straight line = supplementary = add up to 180 degrees
- Angles that share the same center point, and you can draw a circle around, add up to 360 degrees!

Circles:

Degree Measurements:

- A circle contains 360 degrees, which is EQUAL to 2π radians.
- A **central angle** is formed by 2 radii and a center point.
    - The measure of the central angle will be the same as the measure of the intercepted arc.
- An **inscribed angle** is formed by two chords in a circle with a common endpoint.
    - The measure of the inscribed angle will be half the measure of the intercepted arc

Formulas

- Radius = 1/2 diameter

- Circumference = 2πr or 2πd
    - "Revolutions" refer to the circumference (i.e. 5 revolutions = 5 times the circumference)
- Area = πr^2
- Remember always write out the formulas before substituting!

Equation of a Circle: (x-h) 2 + (y-k) 2 = r 2 --> (h,k) is the center of the circle and *r* is the radius

- (h,k) is the center of the circle and *r* is the radius
- When being asked about the equation of a circle write your own equation first, then look to see which multiple choice options match it!

Completing the square

- Besides having to use this method to convert from standard form to vertex form of a parabola, you may also have to complete the square for circle problems.
- Follow the same procedure, except you will have to "complete the square" twice --> once for the x and once for the y.
- Simplify the given equation, so that the coefficient of the variables with a degree of 2 is 1.

Sectors & Arc Lengths of Circles:

- When dealing with a portion of a circle always think PART/WHOLE. Set up a proportion!
- Figure out if you are working with circumference (arc length) or area (sector area).
- The first ratio of the proportion will be the ratio of the sector or arc length over the whole area/circumference. The other side of the proportion will usually be the angle measurement over 360.
- You can also find sectors or arc lengths of circles using the "fractional idea". For example, if I know that the sector of the circle, I am finishing is 60 degrees than that is 1/6 of a circle (or 1/6 of the area or 1/6 of the circumference)!

Tricks for Circle questions on the SAT

- The SAT loves to give you information that will require you to do something with a radius - like give you a square inside a circle, with the side length of the square being the radius.
- When you see semi-triangles inside circles, draw radii and/or connect the radii to make triangles.
- Always think special triangles --> 45/45/40 or 30/60/90 triangles. Think of these triangles when you see squares in circles too!
- Percent of a circle --> percent means "out of 100" so --> can make proportions
- Always use 360 degrees as the total degrees of a circle

Conjugates/complex conjugates

- In algebra, the conjugate is where you change the sign in the middle of the two terms
- (ex: 2x+1 --> 2x - 1)
- A conjugate is especially helpful with fractions, when you want to change or simplify the denominator
    - ex: there may be a square root in the denominator (remember square roots in the denominator is not a fraction in simplest form)
    - ex: want to get simplify/get rid of imaginary numbers in the denominator (example: if the denominator is (3 - 2i), we multiply the fraction by the conjugate of the denominator (3 + 2i), then the "i" would disappears because i2 = -1

Data Analysis

Charts and Tables

- Often used to give an organized picture of information, or data.
- Understand what is given.
- Column headings and line items show important information. These titles give the numbers meaning.

Graphs

Bar Graphs

- Convert the information in a chart into separate charts or columns.
- Determine the relationship between the columns in the graph.

Line Graphs

- Convert data into points on a grid. These points are then connected to show a relationship among the items, dates, and times, for example.
- Can show trends, or changes, in data over a period of time.
- Pay attention to the slopes of these lines. The lines show increases and decreases.
- The sharper the slope upward, the greater the increase.
- The sharper the slope downward, the greater the decrease.

Circle Graphs or Pie Charts

- Shows the relationship between the whole circle (100%) and the various slices that represent portions of that 100%.
- The larger the slice, the higher the percentage.

Scatterplot (Best Fit Modeling)

- A graph representing a set of data and showing a relationship or connection between two quantities given.
- If the points appear to form a line, a linear relationship is suggested.
    - positive association slants to the right
    - negative association slants to the left
- Use corner of your paper to draw a line, that best shows the trend of the points, the" line of best fit".
- Notice scale on the axis. Sometimes the x and y axes' grids aren't measured in a unit of 1.

Box and whiskers (or box plot): displays the five-number summary of a set of data. The five-number summary is the minimum, first quartile, median, third quartile, and maximum.

- In a box plot, we draw a box from the first quartile to the third quartile. A vertical line goes through the box at the median. The whiskers go from each quartile to the minimum or maximum.
- The five-number summary divides the data into sections that each contain approximately 25% of the data. The minimum to Q1 is the 1st 25% of the data. Q1 to Q2 is the second 25% of the data, Q2 to Q3 is the third 25% of the data and Q3 to the maximum is the last 25% of the data.

Charts and Tables - Things to Remember

- Examine the entire graph, noticing labels and headings.
- Focus on the information given. Look for major changes - high points, low points, and trends.
- Don't memorize the chart, table, or graph; refer to it.
- Skimming questions can be helpful.
- Circle or underline important words in the question.
- Pay special attention to which part of the chart, table, or graph the question is referring to.
- If you don't understand the graph, reread the labels and headings.

Exponents Rules

It's all about the Base

- You cannot apply Exponent Rules to Exponents with different bases

- Example: I can use the Exponent Rules for 43 x 45 but not for 43 x 54.
- You cannot add or Subtract bases
- Example: 23 + 23 is *not equal* to 26, however you can re-write addition using multiplication: 23 + 23 = 2x23.

3 Rules You Must Know

1. When you multiply the same bases, you **add** the exponents (23 x 23 = 26).
2. When you divide the same bases, you **subtract** the exponents (23 / 23 = 20).
3. When you raise a power to a power, you **multiply** the exponents (23 x 23 = 26).

Exponential Functions Graphically

- $y = ab^x$ where a is the starting value and b is the base (or multiplier), x is the time!
- A curved graph where the y-intercept is the starting value "a"
- Unlike the linear graph, exponential graphs express growth or decay using multiplication. Values will "halve" or "double".

Interest Rate Formula: $A = P(1 + r/n)^{nt}$ --> if n = 1 --> $A = P(1 + r)^t$

- A = total amount of $
- P = Principal or Initial Amount of $ invested
- r = interest rate (usually given as a percent --> remember to turn to a decimal)
- t = time in years
- n = # of times interest is compounded per year

Functions: a way to describe the relationship between inputs and outputs, whether in graph form or equation form.

- It may help to think of functions like an assembly line or like a recipe--input eggs, butter, and flour, and the output is a cake.
- Most often you'll see functions written as f(x)=an equation
- You can have "many to one" (many inputs to one output), but NOT "one to many" (one input to many outputs).
- This means that a function graph can have potentially many x-intercepts, but only one y-intercept. (Why? Because when the input is x=0, there can only be one output, or y value.)
- input/output → ordered pair/coordinates
- f(x) = g(x) = y
- Functions can always be graphed, and different kinds of functions will produce different looking graphs.

Function Translations (shifts) & Transformations (stretching & shrinking):

- All the rules (with pictures) are in the "review" pages of your SAT Math Workbook. In particular, these are the important rules that you must know:
  - $f(x) = x^2 + c$ (graph moves up) and $x^2 - c$ (graph moves down)
  - $f(x) = (x+c)^2$ (graph moves to the left) and $(x-c)^2$ (graph moves to the right)
  - $f(x) = -(x)^2$ is a reflection in the x-axis ("upside down U")

Evaluating Functions: just substitute the value they are giving you for x! Note, sometimes it may be a variable, but that's ok, just use the same process!

- Example #1: f(r) = (r + 1); what is f(4)
  - f(4) = (4 + 1) = 5
- Example #2: f(r) = (r + 1); what is f(h-4)
  - f(h - 4) = (h - 4 + 1) = (h-3)
- **Composition of Functions:** In order to solve these types of questions, think of them in terms of your order of operations. You must always work from the inside out, so you must first find the output for your innermost function.

Geometry word problems:

- Draw them out!
- 3-D Geometry --> think either volume or surface area
- Remember surface area is area in 3-D, meaning you need to find the area of all the sides and add them up!
- If given information for 3-D figure but only need to "fill up an area" only 2 of those 3 dimensions will matter. To figure out which 2 dimensions to use, look at the question (ex: what is the minimum about of cubes I can fit, etc.)
- If volume/SA (or side lengths) is/are doubling/tripling --> do the problem picking numbers more than once
- The height of a triangular pyramid doesn't form a right angle with the base!

Imaginary Numbers:

- The **complex plane** or z-**plane** is a geometric representation of the **complex numbers** established by the real axis (x-axis in the coordinate plane) and the perpendicular **imaginary** axis (what we know as the y-axis in the coordinate plane).
- $a + bi$ is the standard representation of an imaginary number
- $i^2 = -1$ (Therefore i = the square root of -1)
- Your calculator can simplify imaginary numbers for you, as long as you remember to:
- Put your calculator in "a +bi" mode
- Make sure you have a number for a and b. If you are given variables, for a and b, then remember to pick numbers!

Percent Increase/Decrease

- When being asked about percent increase or decrease--> you are looking for the change in percent, from what you originally started with to what you ended with.
- **Formula**: (new# - old#)/old# ***OR*** |change|/original

Probability:

- the likelihood that a particular event will occur
- It is typically expressed as a fraction; a value from 0 to 1 (0%-100%)
- 1 --> it's a sure thing; 0 --> it's just not going to happen
- "What is the probability?" --> The simple formula: # of outcomes that meet the requirements / Total # of possible outcomes
- "or" --> add
- "and" --> multiply
- These will usually be given in charts on the SAT. Pay close attention to the phrases in the question. Ask yourself what pool of people they are choosing from and what fraction of those people are the part.

Quadratics/Parabolas:

Factoring:

- There are many ways to factor, depending on the problem you are presented with (see page 58 in your Math SAT Workbook).
- DOTS (difference of two squares)
- FOIL method
  - and/or "AC Method"
- GCF (factoring out the GCF first)
- Quadratic formula

Solving:

- You will be asked for solutions, x-intercepts, roots or zeros of an equation.
- Always set the equation equal to 0 before you solve.
- You can use any of the above options for factoring to solve a quadratic.

Standard form of a parabola: $ax^2 + bx + c$ where c is the *y-intercept*

- Be careful, do not use standard form to find the vertex. This form gives you the roots (or x-intercepts) where the graph crosses the x axis. NOT the vertex.
- **Roots** - **Solutions** - **Zeros** are all names for the "**x-intercepts**". Find these by factoring and solving for x.
- The formula **x=-b/2a** gives you the axis of symmetry (the x value of the vertex)
- The formula for the sum of the roots and product of the roots is as follows (please note these formulas are not as important to know):
- sum of the roots: -b/a
- product of the roots: c/a

Vertex Form of a Parabola: F(x) = a(x - h)2 + k where (h,k) is the vertex

- If the problem is in the calculator section, graph the equation to find the turning points or vertex.
- You may have to **complete the square** to convert from standard form to vertex form.

The discriminant: Given a quadratic equation, the discriminant will always tell you if your quadratic equation has No solution, 1 solution or 2 solutions. This formula is a great time saver!

- Given a quadratic in standard form: ax2 + bx + c = 0, the discriminant is found by using the formula: b2 - 4ac.
  - If b2 - 4ac = 0, there is 1 solution
  - If b2 - 4ac = a positive number, there are 2 solutions
  - If b2 - 4ac = a negative number, there are NO solutions

Parabola's Graph:

- All the rules (with pictures) are in the "review" pages of your SAT Math Workbook.
- In particular, these are the important rules that you must know:
  - $f(x) = x^2 + c$ (graph moves up)
  - $x^2 - c$ (graph moves down)
  - $f(x) = (x+c)^2$ (graph moves to the left)

- $(x-c)^2$ (graph moves to the right)
- $f(x) = -(x)^2$ is a reflection in the x-axis ("upside down U")

Standard deviation: How far each data value is from the mean

- the more spread apart the data, the greater the standard deviations
- On the calculator::
  - STAT --> 1 (selects the list-edit screen)
  - Cursor onto the label L1 at the top of the first column, then: CLEAR --> ENTER (this erases the list. Next, enter your values!
  - STAT --> [▶] --> (1) --> ENTER
  - The standard deviation is the "Sx" data

Systems of Equations

There are several **ways to solve** system of equations:

- **Graphing**
  - Solution is where lines (or curves) intersect.
  - Use your calculator to graph. Don't use tables because it won't show you fractions, instead use Trace, (2nd Calc)Intersect
- **Substitution**
  - Solve for one variable and plug that into the other equation. Now you have ONE equation and ONE unknown so you can solve.
- **Elimination**
  - Multiply one equation by a negative constant so when you add the two equations, you "ELIMINATE" one of the variables

Please remember when deciding whether to use "substitution or elimination", look at the problem. If it is set up to make it easy to "eliminate", then use elimination to only leave the variable you are trying to solve for.

It's helpful to **determine the graph** of each equation when solving system of equations:

- Line is $y = mx+ b$
- Parabola is $y = x^2 +bx + c$
- Circle is $x^2+y^2 = r^2$ where r is the radius

Here's **how to determine how many solutions** there are for a system of equations (when both are lines):

- Same slope – no solution – parallel lines
- Different slope – one solution
- Same line (same slope) – infinite solutions

Inequalities:

- For graphs:
  - The symbol $\leq$ tells you to shade down
  - The symbol $\geq$ tells you to shade up
- You can also always pick a point in the solution set & plug it into both systems. If both statements are true, then that point is the solution to the systems!

Statistics

Designing & Interpreting Experiments & Studies

- The more randomized an experiment, the stronger the conclusion that can be drawn.
- If you want to be able to draw conclusions about an entire population from survey results, you need to randomly select a sample from that entire population.
  - In an experiment, you can only generalize results to an entire population if you selected the experimental sample
- The bigger the sample size, the more likely it is that the sample is representative of the whole population.
  - Rule of thumb: if your sample size is 100 or bigger, it's probably fine.
- **Confidence Interval:** included to acknowledge that even if the study was designed carefully, it's mathematically possible that the sample just won't represent the overall population well.

- **Margin of error:** measures the maximum amount by which the sample results are expected to differ from those of the actual population.
- **Standard deviation** = how spread the data values are from the average
    - The more spread apart the data, the stronger the standard deviation.
    - One standard deviation is equal to a +/- range from your mean.

Total Value:
These are a bit challenging they require you to create two different equations in order to solve them. Then, you solve the "simultaneous equations".

- These problems give you TWO total values and TWO variables
- These are generally two different ways to solve these problems: elimination and substitution.

Triangles:

- 3 sides will form a triangle if **a + b > c** (the sum of the lengths of the 2 legs > the length of the hypothenuse)
- **Pythagorean Theorem** --> can be used to find the missing side lengths of right triangles.
- **Area:** area = (bh) /2
- Given a right triangle, the sin(x) = cos(90-x) & the cos(90) = sin(90-x)!

Similar triangles: have congruent angles and proportional sides

- Make sure your ratios are matching the corresponding sides
- Remember --> If two triangles share a same angle or a same side, then that means that angle or side is congruent in both similar triangles
- **Isosceles triangle:**
    - Has two congruent sides.
    - Has congruent base angles.
    - Has an altitude which: (1) meets the base at a right angle, (2) bisects the apex angle, and (3) splits the original isosceles triangle into two congruent halves.

30-60-90 triangle: The ratio of side lengths in such triangles is always the same: if the leg opposite the 30 degree angle is of length x , the leg opposite the 60 degree angle will be of x radical 3 , and the hypotenuse across from the right angle will be 2x .

45-45-90 triangles (also often called isosceles right triangles): If the legs opposite the 45-degree angles are of length x, the hypotenuse has a length of x radical 2.

Unit Conversions:

- Proportions are built from ratios. A" ratio" is just a comparison between two different things.
- For instance, someone can look at a group of people, and refer to the "ratio of men to women" in the group. Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is 15 to 20. Notice that, in the expression "the ratio of men to women", "men" came first. This order is very important, and must be respected: whichever word came first, its number must come first. If the expression had been "the ratio of women to men", then the numbers would have been "20 to 15". Also, can use # s from boys and girls in the class.
- The SAT loves to trick you by giving information in one unit and asking you for an answer in a different unit. Be careful that you are giving your answer in the proper units. For example, you may be given the sides of a rectangle are 7 feet and 5 feet and asked to find the area in INCHES.
- It's easier to do your conversions first before calculations. If asked for area in inches and given a square has a side of 4 ft. Convert 4ft to 48 inches FIRST, then calculate area.
- Proportions have to be EQUAL on both sides of the equation (i.e. the units have to be the same and put in the same positions
- If there is no number after the word PER, then the value is 1
- Ex: 10 dollars per week means $10/week

Dimensional Analysis:

- Uses the fact that any number or expression can be multiplied by one without changing its value.
- A method used to avoid having to make multiple proportions.

- Unit factors may be made from any two terms that describe the same or equivalent "amounts" of what we are interested in.
- For example: I know 1 foot = 12 inches. This ratio can be written 2 ways:
- 1 foot/12 inches or 12 inches/1 foot
- Set up each problem by writing down what you need to find with a question mark. Then set it equal to the information you are given. The problem is solved by multiplying the given data and its units by the appropriate unit factors so that only the desired units are present at the end.

Y = mx + b word problems

- Linear (We know b/c the exponent is 1)
- *b* = y-intercept
  - constant
  - fixed amount, usually seen as "initial value", "base salary", etc.
- *m* = slope
  - constant rate of change
  - slope will be followed by the words "per", "every" or "each"

Y=mx + b - Important things to Remember:

- When translating word problems, sometimes, you will have to calculate for slope before you can substitute it in the slope-form equation.
- When going "under" or "below sea level" slope will be a fraction!
- When you are given an equation, and asked what will happen if you increase a variable by a certain number, the best strategy for this question type is to:
  - First set the equation equal to 0.
  - Then to set the equation equal to the number they want you to increase it by.
  - Then solve both equations.
  - Subtract the values!

Various WORD PROBLEMS on SAT:

- When given a distance time and/or a rate --> think: **d=rt** word problems
- **y = mx+ b** word problems --> key words are "per", "each", "every" and there will always be a starting value.
- Read carefully, sometimes you may have to subtract from the slope or the y-intercept before you plug in!
- **Total value** word problems --> there will be 2 totals --> key to set up systems equations
- **Unit conversions** word problems --> will be given different units in the word problem.
- In a multi-step word problem, convert before you compute!
- Draw out diagrams, esp. for difficult word problems!
- **Percent Change** = change/original or "NOO"
- **Mixture Problems:** these are percent equation problems (involving mixing some sort of liquid)
  - Remember to put these equations in this format: % (liquid 1) + % (liquid 2) = % (liquid 1 + liquid 2)