ACT MATH TOPIC SKILLS

Absolute Value Graphs:
- Since this type of function is always taking the absolute value of the input, the "output" (the y-value) can never be negative therefore, the graph will never be in Quadrants III and IV.
- Pay attention to the degree of the function. It will tell you whether or not the function will look like a "typical" absolute value function.
    - For example: If the degree of the function is 2, you should expect a combination of a parabola/absolute value graph.

Angles:
Angle Definitions:
- **Supplementary Angles:** two angles with a sum of 180 degrees. They usually lie on a straight line but doesn't always needs to be the case.
- **Complementary Angles:** two angles with a sum of 90 degrees. They usually form a right angle, but it doesn't always need to be the case.
- **Vertical Angles:** each of the pairs of opposite angles made by two intersecting lines.
- **Bisector** of an angle: a ray whose end point is the vertex of the angle and which divides the angle into two equal angles.

Transversals: When parallel lines get crossed by another line (called a transversal), some angle relationships are formed:
- **Corresponding angles:** All angles that have the same position with regards to the parallel lines and the transversal are corresponding pairs. They are congruent!
- **Alternate interior angles:** angles on opposite sides of the transversal and on the interior of the parallel lines. These angels are congruent! (You can trace a "Z" around these angles).
- **Alternate exterior angles:** angles on opposite sides of the transversal and on the exterior of the parallel lines. There angles are also congruent!
- **Consecutive angles** may be defined as two interior/exterior angles lying on the same side of the transversal cutting across 2 parallel lines.

Note: This holds true for any type of diagram, where there are parallel lines. For example, sometimes the parallel lines may be the two sides of a rhombus, and the "Transversal" may be the diagonal!

Angles in a Polygon: The sum of the degrees of the angles in a polygon can be found:
- using the formula 180(n - 2,) where n = the number of sides the given polygon
- by breaking the polygon into shapes you know (ex: triangles or quadrilaterals)

Remember --> when you are given a question asking about an angle in a polygon, remember first you must find the total degrees in that given polygon, then you must split the total degree measurement evenly amongst the # of angels in the polygon.

Angles - Reminders
- Vertical angles = opposite angles = congruent
- Angles that lie on a straight line = supplementary = add up to 180 degrees
- Angles that share the same center point, and you can draw a circle around, add up to 360 degrees!

Arithmetic and Geometric Sequences:

- *Arithmetic*: An arithmetic sequence goes from one term to the next by always adding (or subtracting) the same value.
  - The number added (or subtracted) at each stage of an arithmetic sequence is called the "*common difference*", because if you subtract (that is, if you find the difference of) successive terms, you'll always get this common value.
- *Geometric:* A geometric sequence goes from one term to the next by always multiplying (or dividing) by the same value.
  - The number multiplied (or divided) at each stage of a geometric sequence is called the "common ratio", because if you divide (that is, if you find the ratio of) successive terms, you'll always get this common value.

Asymptotes:

- To find the **vertical asymptote(s)** of a rational function: Set the denominator equal to 0 and solve for x.
- To find **horizontal asymptotes**:
  - If the degree (the largest exponent) of the denominator is bigger than the degree of the numerator, the horizontal asymptote is the x-axis (y = 0).
  - If the degree of the numerator is bigger than the denominator, there is no horizontal asymptote.
  - If the degrees of the numerator and denominator are the same, the horizontal asymptote equals the leading coefficient (the coefficient of the largest exponent) of the numerator divided by the leading coefficient of the denominator

Circles:
Degree Measurements:

- A circle contains 360 degrees, which is EQUAL to 2π radians.
- A **central angle** is formed by 2 radii and a center point.
  - The measure of the central angle will be the same as the measure of the intercepted arc.
- An **inscribed angle** is formed by two chords in a circle with a common endpoint.
  - The measure of the inscribed angle will be half the measure of the intercepted arc.

Formulas

- Radius = 1/2 diameter
- Circumference = $2\pi r$ or $2\pi d$
  - "Revolutions" refer to the circumference (i.e. 5 revolutions = 5 times the circumference)
- Area = πr^2
- Remember always write out the formulas before substituting!
- The equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$
  - (h,k) is the center of the circle and *r* is the radius
  - When being asked about the equation of a circle write your own equation first, then look to see which multiple choice options match it!

Sectors & Arc Lengths of Circles:

- When dealing with a portion of a circle always think PART/WHOLE. Set up a proportion!
- Figure out if you are working with circumference (arc length) or area (sector area).
  - The first ratio of the proportion will be the ratio of the sector or arc length over the whole area/circumference. The other side of the proportion will usually be the angle measurement over 360 degrees.

Things to Remember about Circle questions on the ACT

- The ACT loves to give you information that will require you to do something with a radius - like give you a square inside a circle, with the side length of the square being the radius.
- When you see semi-triangles inside circles, draw radii and/or connect the radii to make triangles.

o   Always think special triangles --> 45/45/40 or 30/60/90 triangles. Think of these triangles when you see squares in circles too!

Common Denominators (especially w/algebraic expressions):

- Remember, the "non-mathematical" definition of a common denominator is: "a feature shared by all members of a group". Therefore, when looking for common denominators, we discovered it is easier for you to treat looking for "common denominators (common prime factors)" as looking for "common features of the groups". Each factor of both groups (denominators) will be a factor of the group (new "common denominator"). However, when you are listing the common factors, (or features of both groups), you do not count or state the factors that are common twice, you only state them once. You don't want to repeat yourself.
- Examples: Find the common denominator for:
  o $1/6 + 1/8$
- $= 1/(2 \times 3) + 1/(2 \times 2 \times 2) = 1(2 \times 3 \times 2 \times 2) = 1/24$
  o $1/(x - 2)^2 + 1/(3x - 6)$
- $= 1/(x - 2)(x - 2) + 1/3(x - 2) = 1/3(x - 2)(x - 2) = 1/3(x - 2)^2$

Conic Section: the face formed where a plane intersects a right circular cone. The outline may be a circle, an ellipse, a parabola or a hyperbola. In the special case when the plane touches the vertex, the outline may be a point, line or 2 intersecting lines.

- The shape is a circle when a plane intersects a cone at right angles to the cone's axis.
- The shape is an ellipse when a plane is inclined to the axis and cuts all the generators. (a generator is a straight line which runs from the apex of the cone to the base)
- The shape is a parabola when the plane is parallel to a generator, and only one of the cones.
- The shape is a hyperbola when the cutting plane is inclined at an angle greater than the exterior angle of the cone.

Conjugates/complex conjugates

- In algebra, the conjugate is where you change the sign in the middle of the two terms
- (ex: 2x+1 --> 2x - 1)
- A conjugate is especially helpful with fractions, when you want to change or simplify the denominator
  o   ex: there may be a square root in the denominator (remember square roots in the denominator is not a fraction in simplest form)
  o   ex: want to get simplify/get rid of imaginary numbers in the denominator

Coterminal Angles:

- same angles with different measures; have the same initial and terminal sides
- To find coterminal angels:
  o   In degrees, add or subtract 360
  o   In radians, add or subtract 2π

Counting:
Factorial

- $n!$ (n factorial): a simple way of writing the product of all the positive whole numbers up to a given number (from 1 to n)
  o   In your calculator: math --> PRB --> !
- **Example:** How many ways can we arrange 5 boxes of cereal on a shelf: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Permutations

- The number of permutations of n distinct objects taken r at a time, denoted by $_nP_r$ where repetitions are not allowed.

- the order that we arrange the objects in *is* important
- In your calculator: math --> PRB --> $_nP_r$
- **Example:** How many ways can we arrange 5 boxes of cereal in 3 spaces on a shelf: $_5P_3 = 60$

Combinations
- The number of ways (or combinations) in which *r* objects (groups) can be selected from a set of *n* objects, denoted by $_nC_r$ where repetition is not allowed.
- the order that we arrange objects in *is not* important.
- In your calculator: math --> PRB --> $_nC_r$
- **Example:** How many ways can 3 boxes of cereal be selected from a batch of 20 different boxes of cereal: $_{20}C_3 = 60$.

Counting- Important things to Remember:
- Factorial is a permutation.
- A permeation is a limited factorial.
- Order matters for permutations, but not combinations.
- "Different groups" or "committees" are key words for combination word problems.

Digits: Predicting Digits in a Repeating Decimal
- Step 1: Count how many digits there are in the repeating decimal (ex: in 0.285714, there are 6 digits). This is called the period.
- Step 2: Divide the Digit # you are trying to find by the period (ex: if asked to find the 1,001st decimal place of 0.285714, divide 1,001/6)
- Step 3: Look at your remainder (In the given quotient above, the remainder is 5). Your remainder tells you this given Digit number will end at the remainder's position number (For example the remainder 5 tells me that the 1,0001st digit will be the 5th number in the given decimal: 0.285714).

Direct and Inverse Variation:
Direct Variation (equations we are used to seeing):
- "y varies directly with x" or both *x* and *y* increase or decrease
- A relationship between two variables in which one is a constant multiple of the other. In particular, when one variable changes the other changes in proportion to the first. If y is directly proportional to x, the equation is of the form   y = kx (where k is a constant)

Indirect Variation:
- "y varies indirectly with x" or one variable (x or y) increases while the other one (x or y) decreases in proportion so that the product isn't changed.
- A relationship between two variables in which the product is a constant. When one variable increases the other decreases in proportion so that the product is unchanged. If y is **inversely** proportional to x, the equation is of the form y = k/x (where k is a constant). Example: y = 60/x or y = 60 times 1/x.

Ellipses: It is basically an oval (or squished circle)
- The equation of an ellipse is similar to that of a circle: $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$
    - a = horizontal radius & b = vertical radius
    - This is not the only way this formula may be written. But all you need to remember is that the denominator with the x-axis is the radius in the horizontal direction and the denominator with the y-axis part of the formula is the radius in the vertical direction!
    - (h,k) is the center of the ellipse --> if you don't see a number next to h and k, the center is (0,0).

Foci of an Ellipse
- 2 points whose sum of distances form any point on an ellipse is always the same
- lie of the major axis (radius)
- To find the foci, use the formula: c2 = a2 - b2
    - a = the larger radius

- o   b = smaller radius
- o   c = distance from center of the ellipse to point of foci

Expected value = the predicted variable, calculated as the sum of all possible values, each multiplied by the probability of its occurrence.

Exponents Rules
It's all about the Base
- You cannot apply Exponent Rules to Exponents with different bases

  - o   Example: I can use the Exponent Rules for 43 x 45 but not for 43 x 54.
- You cannot add or Subtract bases
  - o   Example: 23 + 23 is *not equal* to 26, however you can re-write addition using multiplication: 23 + 23 = 2x23.

5 Rules You Must Know
- When you multiply the same bases, you **add** the exponents (23 x 23 = 26).
- When you divide the same bases, you **subtract** the exponents (23 / 23 = 20).
- When you raise a power to a power, you **multiply** the exponents (23 x 23 = 26).
- When re-writing a racial to a fractional exponent: $m\sqrt{(a\ n)} = a\ n^/m$ [Ex: 2√(26) = 26/2 = 23]
- Negative Exponents Rule: b-n = 1 / bn [Ex: 2-3 = 1 /23]

Exponential Functions Graphically
- $y = ab^x$ where *a* is the starting value and *b* is the base (or multiplier)
- A curved graph where the y-intercept is the starting value "*a*"
- Unlike the linear graph, exponential graphs express growth or decay using multiplication. Values will "halve" or "double".

Functions:
Evaluating Functions: To evaluate a function --> substitute its variable with a given number or expression
Composition of Functions:
- "Function Composition" is applying one function to the results of another
- The notation used for the composition of functions looks like this, f(g(x))
  - o   This function is read as "f of g of x"
- In composite functions it is very important that we pay close attention to the order in which the composition of the functions is written. In many cases (f(g(x)) is not the same as g(f(x)).
- Work from "in to out" (see steps below)
- Steps we can use to find the composition of two functions:
  - o   Step 1: Replace each occurrence of x found in the outside function with the inside function. For example, in the composition of (f(g(x)), we need to replace each x found in f(x), the outside function, with g(x), the inside function.
  - o   Step 2: Simplify the answer.
- Example: If f(x) = –4x + 9 and g(x) = 2x – 7, find f(g(x)).
  - o   Step 1: Replace each occurrence of x in f(x) with g(x). Basically --> the new "x" in f(x) is g(x) --> f(g(x))= -4(2x - 7) + 9
  - o   Step 2: Simplify the answer by distributing and combining like terms -->f(g(x)) = -8x+28+9 = 8x+37

Odd & Even Functions (graphically):
- **Even Function**: If the graph is "symmetric about the y-axis" (whatever the graph is doing on one side of the y-axis is mirrored on the other side)
- **Odd Function**: If the graph is "symmetric about the origin" (half of the graph on one side of the y-axis is the upside-down version of the half of the graph on the other side of the y-axis).

Geometry word problems:
- Draw them out!
- Sometimes looking at the multiple-choice options gives you a clue about how to go about answering the question.
- 2-D Geometry --> think area or perimeter
- 3-D Geometry --> think either volume or surface area
- To find the total degree measurement of any polygon, first cut the shape into shapes you know. Add up the total degrees of each of the shapes. Then divide the total degrees by the number of angles. Your answer will tell you how many degrees each angle is.

Imaginary Numbers:
- The **complex plane** or z-**plane** is a geometric representation of the **complex numbers** established by the real axis (x-axis in the coordinate plane) and the perpendicular **imaginary** axis (what we know as the y-axis in the coordinate plane).
- $a + b$i is the standard representation of an imaginary number
- $i^2 = -1$ (Therefore i = the square root of -1)
- Your calculator can simplify imaginary numbers for you, as long as you remember to:
  - Put your calculator in "a +bi" mode
  - Make sure you have a number for a and b. If you are given variables, for a and b, then remember to pick numbers!

Imaginary Numbers: **Distance (in the complex plane)**
- To find the distance in the complex plane between two imaginary numbers, you can still use the distance formula (by hand or on your calculator). Use the same process as you would with real-number coordinates, but ignoring the "i" which stand for the coordinate on the imaginary axis
  - Example: Find the distance between 2 + 6i .and -4 + 3i
    - x2 = -4, y2 = 3 and x1 = 2 and y1 = 6

Inequalities
- If given to you in the format of an "equation" (not a graph) to solve for x, on the ACT, always treat these as an equation!! For example, if it tells you that x is between 1 and 4, use 1 and 4!
- Please note, you don't ever need to memorize the rules below, because you can always pick a point & plug it into the given equations. If both sides of the equation are true → that's the answer!

Line Graphs
- The symbol $\leq$ or < tells you to shade down
- The symbol $\geq$ or > tells you to shade up

Circle graphs
- The symbol $\leq$ or < tells you to shade inside the circle
- The symbol $\geq$ or > tells you to shade outside the circle

Parabola graphs
- The symbol $\leq$ or < tells you to shade outside the parabola
- The symbol $\geq$ or > tells you to shade "inside" the parabola

Interest Rate Formula: $A = P (1 + r/n)^{nt}$ --> if n = 1 --> $A = P (1 + r)^t$
- A = total amount of $
- P = Principal or Initial Amount of $ invested
- r = interest rate (usually given as a percent --> remember to turn to a decimal)
- t = time in years
- n = # of times interest is compounded per year

Inverse Functions:
- a function that "reverses" another function
- written as $f(x) = f^{-1}(x)$
- To find the inverse
  - solve the original function for x
  - then switch y and x

Least Common Multiple:
- You don't need to do an LCM problem by hand!! It is too time consuming! On the calculator: "MATH" --> Scroll to the right to "Num" --> scroll down to "8:lcm("
- Remember the calculator only allows you to take the LCM of 2 numbers at a time. There is a way to bypass this.
  - For example, when you have to take the LCM of 3 numbers, first take the LCM of any of the 2 numbers on your list (order doesn't matter). Then go back into the LCM program and take the LCM of your answer (by choosing ANS on your calculator) and the remaining number.

Computing LCM by hand (i.e. when have algebraic denominators)
- Step 1: List the prime factors of each denominator
- Step 2: Multiply each factor the greatest number of times it occurs in either number.
- If the same factor occurs more than once in both numbers, you multiply the factor the greatest number of times it occurs.
- Easy Example: Find the LCM of 30 and 45
  - *Step 1:* 30 = 2 x 3 x 5 and 45 = 3 x 3 x 5
  - *Step 2:* 2=one occurrence; 3=two occurrences; 5=one occurrence
  - $2 \times 3 \times 3 \times 5 = 90$ <— **LCM**

Logarithms:
- Logarithms can be solved on the calculator for the ACT!!!
- Use the working backwards strategy to solve these, plugging in the answer choices for x in your calculator--> your calculator will do all the work for you!
  - For base 10 problems: use "LOG" button
  - For all Log problems: "MATH" --> Scroll up to "LogBase" OR "ALPHA" --> "Y=" --> "WINDOW" ---> "5:logBASE("
- Some basic logarithm rules:
  - $\log_b(mn) = \log_b(m) + \log_b(n)$
  - $\log_b(m/n) = \log_b(m) - \log_b(n)$
  - $\log_b(mn) = n \times \log_b(m)$
  - Change of bases: $\log_x(a) = \log_x(a) / \log_x(b)$ where "x" can be any number you choose
  - Relationships between Exponential and Logs: $\log_b y = x = b^x = y$

Matrices: A Matrix is a rectangular array of numbers, symbols or expressions.
- Remember that your calculator can help you solve most matrix problems.
  - To access the Matrix program: "2nd" --> "x-1"

Defining/Naming a Matrix: The name of a matrix is the dimensions of a given Matrix --> the given number of rows by the number or columns
- Example: a 2 x 3 Matrix has 2 rows and 3 columns

Augmented Matrix: A matrix representing a system of equations
- Each row represents the constants from one equation (both the coefficients and the constant on the other side of the equal sign)
- Each column represents all the coefficients for a single variable
  - 1st column represents the coefficients of the x's

- o 2nd column represents the coefficients of the y's
- The line between the first 2 columns and the last represents the equal sign of the systems
  - o sometimes the constants are a second matrix instead of being separated by a line

Defined or Undefined Matrix

- Two matrices are defined if you can multiply them. If you cannot they are undefined.
- *Option 1:* plug both matrices in your calculator & multiply. If it says error --> the matrices cannot be multiplied! If you get an answer (a new matrix) --> the matrices are defined (aka they can be multiplied).
- *Option 2:* Check their dimensions!
  - o You must first define each matrix (name them w/their dimensions).
  - o Two Matrices are defined if the # of *columns* of the first matrix match the # of *rows* of the second (the inside numbers) matrix. If this is not the case, the matrix is undefined. If it does work, the answer will look like the outer two numbers (number of rows of the first by number of columns of the second)

Operations with Matrices

- To add/subtract matrices, simply add and subtract the numbers that are in the same position in each matrix.
- Scalar multiplication --> If there is a number in front of the matrix, distribute that number to each number inside the matrix
- To multiply 2 matrices:
  - o Plug into your calculator - no need to multiply by hand!
    - ▪ To access the Matrix program: "2nd" --> "x-1"
    - ▪ Remember that your calculator can help you solve most matrix problems. You must first edit the matrix to enter into your calculator before you do anything else.

Determinant of a Matrix: The determinant tells us things about the matrix that are useful in systems of linear equations, helps us find the inverse of a matrix, is useful in calculus and more.

- The symbol for determinant is two vertical lines either side.
  - o Example: |A| means the determinant of A
- To calculate the determinant, plug it in your calculator! (Sometimes they will give you the rule and you can just follow it)
  - o "2nd" --> "x-1" --> MATH --> 1: det(

Modes of Central Tendency & Range:

- MEAN = average
- MEDIAN = middle # (once data is arranged in order)
- MODE = # that appears the most
- *RANGE* = largest # - smallest #
- **Standard deviation:** How far each data value is from the mean. The more spread apart the data, the greater the standard deviation.
  - o On the calculator:
    - ▪ STAT --> 1 (selects the list-edit screen)
    - ▪ Cursor onto the label L1 at the top of the first column, then: CLEAR --> ENTER (this erases the list. Next, enter your values!
    - ▪ STAT --> [▶] --> (1) --> ENTER
    - ▪ The standard deviation is the "Sx" data

Percents: There are various ways percent questions will be asked. Below are a few of the most common:

- To determine percent of a number, change the percent to a fraction or decimal (whichever is easier for you) and multiply. *Remember* --> The word *of* means to multiply & the word *is* replaces the = when you are translating!
  - o Example: What is 20% of 80?

- Solution: x= (.20)(80) --> Answer: 16
- To find the percent one number is of another, translate the sentence into an equation (please keep in mind this isn't the only method, only the one I personally find the simplest). Simply turn the question word-for word into an equation. You can then solve the equation or if possible, use the multiple-choice options to *work backwards*.
  - Example: 20% of what number is 80?
    - Solution: (.20)(x) = 80 --> Answer: 400
  - Example: 80 is 20% of what number?
    - Solution: 80 = (.20)(x) = 80 --> Answer: 400
- Remember you can always use proportions to solve any percent problems! Just remember these ratios:
  - (%#)/(100) = ("is" number)/("of" number)

Percent Increase/Decrease
- You are looking for the change in percent, from what you originally started with to what you ended with.
- Remember the WilsonPrep acronym "NOO": (New# - Old#)/Old#

Probability:
- the likelihood that a particular event will occur
- It is typically expressed as a fraction; a value from 0 to 1 (0%-100%)
  - 1 --> it's a sure thing
  - 0 --> it's just not going to happen
- "What is the probability?" --> The simple formula: # of outcomes that meet the requirements / Total # of possible outcomes
- "or" --> add
- "and" --> multiply
  - You know you will be multiplying because you will be doing the experiment AND then the problem will tell you that you are doing it again (or multiple times: example 50 times)
  - you may see the word "both" which means you are doing the 1st trial *and* then the other
- S*ample Space*: a set of all the results or outcomes that are possible in that experiment or trial, that set is said to be a sample space of that particular experiment.
  - You can make a list or draw a "tree" diagram.

"At Least" Trick
- 1 - probability of the event not happening

Proportions:
- You can set up a proportion when you have 2 different units in the given word problem.
- Before you plug in numbers for your proportion, set up a ratio with units (what you will be comparing)
- Before beginning to solve, be sure that you only have 2 units, and each unit appears twice in your proportion.
- Remember, a ratio is a fraction. Therefore, equivalent ratios can be represented with a variable.
  - Example: 3 to 4 = 3x to 4x

Properties of Addition and Multiplication:
- **Commutative Property:** Allows you to add or multiply in any order and you will get the same result.
  - Example: 5 + 4+ 10 = 10 + 5 + 4
- **Associative Property**: Allows you to groups terms in any order before you add or multiply and you will get the same result.
  - Example: (2 x 7) x 9 = 2 x (7 x 9)

Set Notation: *A set,* informally, is a collection of things
- Sets are usually named using capital letters

- Sets are "unordered", which means that the things in the set do not have to be listed in any particular order
- All the elements of a set, are everything that is listed in the set.
  - Example: {2, 3, 4, 5} mean that 2, 3, 4, and 5 are part of this given
- If you're going to be technical, you can use full "set-builder notation" to express the above mathematical set. In set-builder notation, the previous set looks like this: $\{x \mid 2 \leq x \leq 5\}$.
  - The above is pronounced as "the set of all x, such that x is an element of the natural numbers 2, 3, 4 and 5". The vertical bar is usually pronounced as "such that", and it comes between the name of the variable you're using to stand for the elements and the rule that tells you what those elements actually are.

Stem-Leaf Plot:
- Another way of arranging data
- a table where each data value is split into a "stem" and a "leaf"
- The "stem" is the first digit or digits
- The "leaf" is usually the last digit
- The "stem" values are listed down, and the "leaf" values go right (or left) from the stem values

Tricks for **Systems of Equation Questions** on the ACT
- No solution = parallel lines = different lines w/same slope
- 1 solution = intersecting lines = different lines w/different slopes
- If intersecting perpendicularly --> negative reciprocal slopes
- Infinitely many solutions or multiple solutions = Same line w/same slope
- manipulate the equations so the coefficients of the x and y values and the constants of both equations are the same.

Trapezoids
Isosceles Trapezoid (properties):
- The bases of an isosceles trapezoid are parallel.
- The opposite sides of an isosceles trapezoid are the same length (congruent).
- The angles on either side of the bases are the same size/measure (congruent).
- The diagonals are congruent.
- Adjacent angles along the sides are supplementary.

Median of a Trapezoid A line segment between the midpoints of the legs of a trapezoid. Also known as midline or mid-segment.
- The median line is always parallel to the bases
- Given a trapezoid, the length of the median is the average length of the bases
- Formula: (base 1 + base 2)/2 = mid-segment in the trapezoid

Triangles:
- 3 sides will form a triangle if **a + b > c** (the sum of the lengths of the 2 legs > the length of the hypothenuse)
- **Pythagorean Theorem** --> can be used to find the missing side lengths of right triangles.
- **Area:** area = (bh) /2
- If the side lengths fit this rule: $a^2 + b^2 = c^2$, we know the triangle if **right.**
- If the side lengths fit this rule: $a^2 + b^2 < c^2$, we know the triangle if **obtuse.**
- If the side lengths fit this rule: $a^2 + b^2 > c^2$, we know the triangle is **acute.**

Similar triangles: have congruent angles and proportional sides

- Make sure your ratios are matching the corresponding sides
- Remember --> If two triangles share a same angle or a same side, then that means that angle or side is congruent in both similar triangles

Isosceles triangle:
- Has two congruent sides.
- Has congruent base angles.
- Has an altitude which: (1) meets the base at a right angle, (2) bisects the apex angle, and (3) splits the original isosceles triangle into two congruent halves.

30-60-90 triangle: The ratio of side lengths in such triangles is always the same: if the leg opposite the 30-degree angle is of length x, the leg opposite the 60-degree angle will be of x radical 3, and the hypotenuse across from the right angle will be 2x.

45-45-90 triangles (also often called isosceles right triangles): If the legs opposite the 45-degree angles are of length x, the hypotenuse has a length of x radical 2.

Trigonometry of Right Triangles: SOH CAH TOA:
- "SOH-CAH-TOA" is only for right triangles. You can use Law of Sines and Law of Cosines on any triangle.
  - SOH stands for the ratio: sin x = opposite/hypothenuse
  - CAH stands for the ratio: cos x = adjacent/hypothenuse
  - TOA stands for the ratio: tan x = opposite/adjacent
- Reciprocal Trig Ratios:
  - CSC stands for the ratio: 1/sin x = hypothenuse/opposite
  - SEC stands for the ratio: 1/ cos x = hypothenuse/adjacent
  - COT stands for the ratio: 1/tan x = adjacent/opposite

SOH-CAH-TOA - Tricks to Remember
- You do not have to read all SOH-CAH-TOA problems - if the diagram is fully labeled
- When given identities (rules) to use you don't recognize, don't worry about it! Just use the rule they give you! That's all they want you to do
- Sin/Cos/Tan on the calculator:
  - When you are solving for a given side, you use the regular function on your calculator. Don't forget when you enter an angle measurement in your calculator, you must be sure that the mode is in degree.
  - When solving for the missing angle, use the inverse function on your calculator ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$)
- "Respectively" means that the given measurements match the given side lengths (or it could be angles) in order!
- Remember the ACT also loves similar triangles. Therefore, if they are giving you an embedded similar triangle in the larger triangle, they want you to use it, to find the missing information! This is especially true when it is question #40 and above!

Trigonometry
- "SOH-CAH-TOA" is only for right triangles. You can use Law of Sines and Law of Cosines (see below) on any triangle.
- To convert radians to degrees, replace the π with 180
- The acronym "All Students Take Calculus" is helpful in determining in which quadrants SIN, COS and TAN are positive/negative.
- Remember when the question is asking about angles measurements, you can usually pick numbers!
- Remember to always use your calculator, if you need to:
  - find an angle measurement (use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ functions)
  - find the ratio, given an angle measurement (use the regular sin, cos, tan functions)
- *Terminal sides* of angles can be found by adding or subtracting 360 degrees from the given angle measurement!

LAWS OF SINES & COSINES: can be used to find a missing angle or side of any triangle!

Law of Sines: States that given any triangle ABC, when we divide side a by the sine of angle A, it is equal to side b divided by the sine of angle B, and also equal to side c divided by the sine of angle C.

- When to use it: When given an angle and 2 sides or 2 angles and 1 side.

Law of Cosines: (also called the **Cosine Rule**) says: $c^2 = a^2 + b^2 - 2ab \cos(C)$

- When to use it: When given an angle and 2 sides or finding any angle of a given triangle when all 3 sides are known.

Trigonometric Functions' Graphs:

- Can be transformed and translated, just like you can transform and translate other functions in algebra.
- *Many times, you can Pick Numbers and Graph these given functions to see the period and amplitude!*
- *Amplitude:* The height of the function. The amplitude just says how "tall" or "short" the curve is. It will be the largest *y* value of that graph.
    - In an equation --> it is the number multiplying the function.
        - For example: y = 3cos(x) --> amplitude is 3
        - For example: y =2sin(4x + 1) + 5 --> amplitude is 2
- *Period:* The distance required for the function to complete one full cycle. It will be the distance between two consecutive maximum points, or two consecutive minimum points (these distances must be equal).
    - In a cosine, sine, css and sec function --> 2π divided by the coefficient of x
        - For example: y = 3cos(x) --> period = $2\pi/|1| = 2\pi$
        - For example: y =2sin(4x + 1) + 5 --> period = $2\pi/|4| = \pi/2$
    - In tan and cot functions --> π divided by the coefficient of x
        - For example: y = 3tan(x) --> period = $\pi/|1| = \pi$
        - For example: y =2tan(4x + 1) + 5 --> period = $\pi/|4| = \pi/4$

Unit Conversions:

The ACT loves to trick you by giving information in one unit and asking you for an answer in a different unit. Be careful that you are giving your answer in the proper units. For example, you may be given the sides of a rectangle are 7 feet and 5 feet and asked to find the area in INCHES.

- Example: IF being asked about the difference (in degrees Fahrenheit) between two given temperatures in degrees Celsius --> convert first!

Vectors: Usually seen in physics, can apply to a lot of things, such as acceleration or velocity

- a vector is a quantity that has magnitude (size) and direction
    - the length of the line shows its magnitude and the arrowhead points in the direction (in a pic - it is a ray)
- vectors can be drawn in the coordinate grid and can also be added graphically.
- we can **add vectors** by adding the x and y coordinates (easiest strategy!)
    - If vectors do not start at (0,0), first plot each vector at the origin.
        - To do this --> $[(x_2-x_1), (y_2-y_1)]$
- vectors can also be drawn in the coordinate grid & can also be added graphically:
    - plot first vector (starting at (0,0) is easiest)
    - Plot second vector (also starting at (0,0) is easiest)
    - Take second vector and change its position and start the second vector at the "tail end" (or arrow) or the second vector.
    - The total distance is now the line that is created from the origin (or the starting point of the first vector) to the "tail end" (or arrow) of the 2nd vector.

Imaginary Numbers as Vectors: an imaginary number is written in the form a + bi

- example: 3 + 4i
- this representation in the complex plane means "3 units along the real axis (x-axis" and 4 units along the imaginary axis (aka the "y" axis)