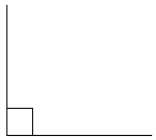
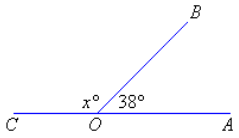
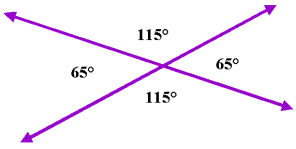
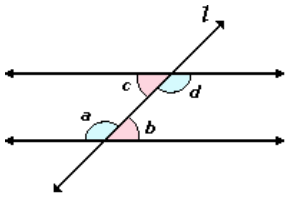
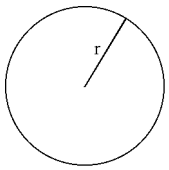
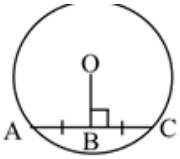
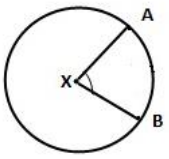
ACT Math Formulas

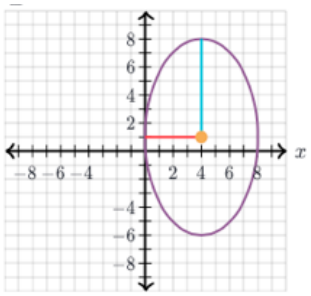
Angles

Right Angles measure 90 degrees	
Supplementary angles add up to 180 degrees	
Vertical angles are congruent	
For parallel lines cut by a transversal, alternate interior angles are congruent	

Circles

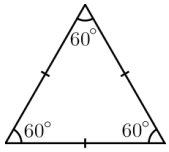
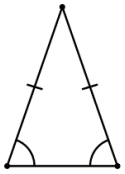
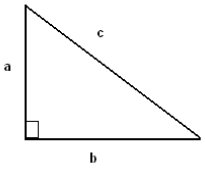
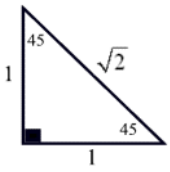
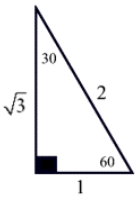
Area A = πr^2 Circumference C = $2\pi r$ Diameter d = 2r	
The perpendicular bisector of a chord in a circle passes through the center of the circle	
Measure of arc AB = x Length of arc AB = $2\pi r \left(\frac{x}{360}\right)$	

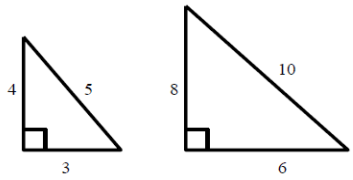
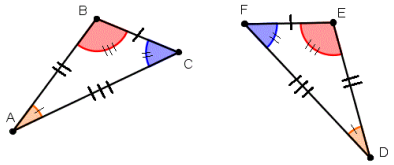
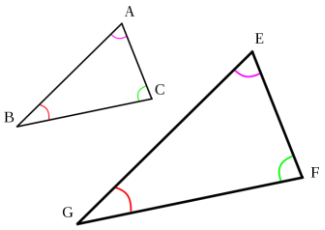
Ellipse: a regular oval shape, traced by a point moving in a plane so that the sum of its distances from two other points (the foci) is constant, or resulting when a cone is cut by an oblique plane that does not intersect the base.

Equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where: (h,k) is the center of the ellipse ***a*** is the radius along the x-axis ***b*** is the radius along the y-axis	*Example:* $\frac{(x-1)^2}{16} + \frac{(y-1)^2}{9} = 1$ 

Triangles:

*For ALL TRIANGLES, the 3 angles add to 180 degrees and Area A= (½) base * height*

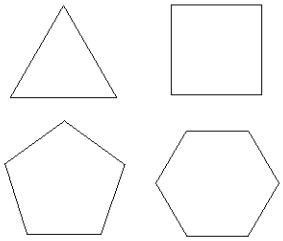
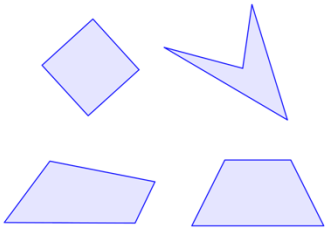
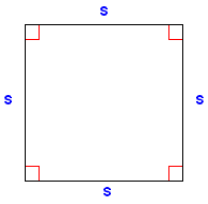
Equilateral Triangle has 3 identical sides and 3 identical angles of 60 degrees (since 60 + 60 + 60 = 180)	
Isosceles Triangle has 2 identical sides and 2 identical angles	
Right Triangle has one 90 degree angle, sides that satisfy $a^2+b^2=c^2$ (Pythag. Thm.), and the other 2 angles are <90 degrees	
45-45-90 Right Triangle has sides with ratio 1:1:√2	
30-60-90 Right Triangle has sides with ratio 1:2:√3 (hypotenuse = 2)	

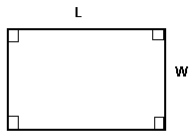
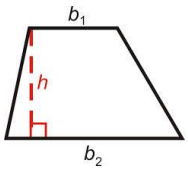
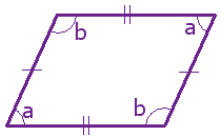
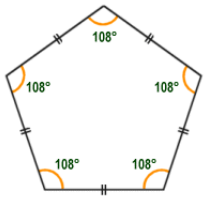
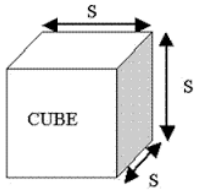
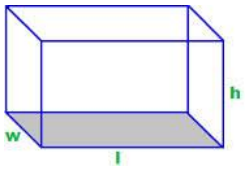
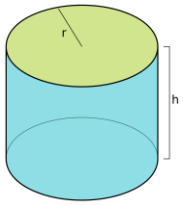
3-4-5 Right Triangle has sides proportional to 3, 4, & 5	
Congruent Triangles have identical side lengths and angles. They may have been rotated or reflected relative to one another and still be congruent.	
Similar Triangles have identical corresponding angles & proportional side lengths	

Polygons

Interior angles of n-sided polygon add up to (n-2)*180

Perimeter of a polygon is the sum of the side lengths

Regular Polygon has equal side lengths and equal angles each measuring $\frac{(n-2)\cdot 180}{n}$	
Quadrilateral is a 4-sided polygon The angles of a quadrilateral sum to (4-2)180 = 360	
Square with side length s Area = s^2 Perimeter = 4s	

Rectangle Area = lw Perimeter = 2l + 2w	
Trapezoid is a quadrilateral with one pair of parallel sides. Find area by dividing into simpler shapes. $A = \frac{b1+b2}{2}h$	
Parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides are equal, opposite angles have equal measure.	
Regular Pentagon is a 5-sided polygon with equal sides and equal angles. The five angles each measure (5-2)180/5=108	
Cube $V = s^3$ $A = 6s^2$	
Box V = lwh A = 2lw + 2lh + 2wh	
Cylinder $V = \pi r^2 h$	

Other Formulas to Know

Negative Exponent: $x^{-n} = 1/x^n$

Raising Power to Powers: Multiply exponents

ex) $(x^2)^3 = x^6$

Multiplying Power Exponents: Add exponents and keep the same base

ex) $(x^2)(x^3) = x^5$

Dividing Power Exponents: Subtract exponents and keep same base

ex) $x^6 / x^4 = x^2$

Factoring Difference of squares

$a^2 - b^2 = (a+b)(a-b)$

FOIL: First, Outside, Inside, Last

ex) $(x-3) - (x+5) =$

$x^2 + 5x - 3x - 15 =$

$x^2 + 2x - 15$

Midpoint Formula: $(\frac{x1-x2}{2}, \frac{y1+y2}{2})$

Percents: Use proportions

ex) 75% of 300

$\frac{75}{100} = \frac{x}{300}$

13 Key ACT Math Concepts

1. Definitions of

   Integer: Negative and positive whole numbers

   ex) …, -3, -2, -1, 0, 1, 2, 3, …

   Rational Number: Real numbers that can be written as simple fractions

   ex) 15, 0.33333..., 7/2

   Irrational Number: Real numbers that cannot be written as simple fractions

   ex) pi (π), 0.35671482...

   Prime Number: A number that can only be divided by 1 or itself

   ex) 2, 13, 71

Imaginary Number: Numbers that when squared give a negative result

ex) $i = \sqrt{(-1)}$, $i^2 = -1$

Undefined Number: A number divided by zero is undefined

ex) 12/0 = undefined, 139/0 = undefined

2. PEMDAS = Order of Operations

Parentheses | Exponents | Multiplication | Division | Addition | Subtraction

ex) $5 * (2+6) – 12 / 6 + 4^2$ Evaluate Parenthesis

$5 * 8 – 12 / 6 + 4^2$	Evaluate Exponents
$5 * 8 – 12 / 6 + 16$	Evaluate Multiplication/Division from left to right
$40 – 2 + 16$	Evaluate Addition/Subtraction from left to right
54	You've found the answer!

3. Operations with Negative Numbers

Adding and subtracting

To add a negative number, *subtract* its opposite

ex) 5 + (-2) = 5 – 2 = 3

To subtract a negative number, *add* its opposite

ex) 11 - (-2) = 11 + 2 = 13

Multiplying and dividing

Multiply or divide with their signs removed

Make the result positive if the total number of negative signs is even

ex) (-5) * (-4) → think: 5 * 4 = 20 → stays positive (result is 20)

Make the result negative if the total number of negative signs is odd

ex) (-3) * (4) →think: 3 * 4 = 12 → becomes negative (result is -12)

4. Divisibility

A number is divisible by 3 if the sum of its digits is divisible by 3

ex) 102 → 1+0+2 = 3 → 102 is divisible by 3

A number is divisible by 4 if the last two digits are divisible by 4

ex) 112 → the last two digits, 12, are divisible by 4 → 112 is divisible by 4

A number is divisible by 5 if the last digit is a 0 or a 5

ex) 95 → the last digit is 5 → 95 is divisible by 5

A number is divisible by 9 if the sum of the digits are evenly divisible by 9

ex) 4518 → 4+5+1+8=18 and 18/9=2 → 4518 is divisible by 9

5. Factors and Multiples

Factors are the numbers we can **multiply together** to get another number

ex) 1, 2, 3, 4, 6, and 12 are factors of 12

Multiples are the result of **multiplying** a number **by an integer** (not a fraction)

ex) 9, 18, 27, 36, 45,... are multiples of 9

6. Remainder: The amount left over after division
   ex) 17/5 has a remainder of 2

7. Absolute Value: |u| = a is the same as u = ±a and vice versa
   ex) Solve | x + 5 | = 12
   x + 5 = ± 12 will have two solutions
   x + 5 = 12 x + 5 = -12
   x = 7 x = -17

8. Finding the missing number in a sequence
   Determine the difference between two numbers, count the number of jumps to get there, and write out several terms
   ex) Find the missing number: 37, 30, 23, ? , 9
   The difference is -7 between each term
   The missing term is 16 (because 23 – 7 = 16)

   * Note: - This tip is for arithmetic sequences
   - Geometric sequences on this test are often very easy {1, 2, 4, 8,16,..} and are typically defined within the question

9. Finding a particular digit in a repeating decimal
   Write out enough digits to see where the pattern repeats.
   Once you determine the pattern, think remainder!
   ex) Find the 55^{th} digit for the decimal 0.15242524...
   This repeats every four digits → Divide 55 by 4 → remainder of 3
   Want the third term of the repeating sequence: The 55^{th} digit is 2

10. Adding and Subtracting Matrices
    In order to add or subtract two matrices, they must be the same size (have the same dimensions: # rows and # columns)

    The rules to add and subtract matrices is below:

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} + \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \\ c_3 & d_3 \end{bmatrix} = \begin{bmatrix} a_1 + c_1 & b_1 + d_1 \\ a_2 + c_2 & b_2 + d_2 \\ a_3 + c_3 & b_3 + d_3 \end{bmatrix}.$$

Similarly, $$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} - \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \\ c_3 & d_3 \end{bmatrix} = \begin{bmatrix} a_1 - c_1 & b_1 - d_1 \\ a_2 - c_2 & b_2 - d_2 \\ a_3 - c_3 & b_3 - d_3 \end{bmatrix}.$$

ex)

$$\begin{bmatrix} 1 & -4 & 5 \\ 2 & 0 & -8 \end{bmatrix} + \begin{bmatrix} -4 & -2 & 1 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} -3 & -6 & 6 \\ 2 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 5 \\ 2 & 0 & -8 \end{bmatrix} - \begin{bmatrix} -4 & -2 & 1 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 4 \\ 2 & -1 & -13 \end{bmatrix}$$

11. Finding the new Mean/Average

To bring the average UP, add a number LARGER than current average

ex) The average of {7, 8, 12} is 9 → the new average of {7, 8, 12, **13**} is **10**

To bring the average DOWN, add a number SMALLER than current average

ex) The average of {7, 8, 12} is 9 → the new average of {7, 8, 12, **5**} is **8**

12. Working with Sets

When determining the number of integers from one to another, be careful about counting inclusively or exclusively. Counting inclusively includes the starting number.

ex) How many numbers are from 1 to 5 inclusive? Exclusive?

Inclusive: 1, 2, 3, 4, 5 → 5 numbers

Exclusive: 2, 3, 4, 5 → 4 numbers

For a set containing evenly spaced numbers, the *mean and median are the same* and equal to the average of the first and last term of the set

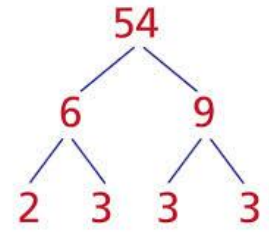
ex) For the set {2, 4, 6, 8. 10}

The mean and median are both 6

Which is also equal to the average of 2 and 10 → (2 + 10) / 2 = 6

13. Prime factorization: Finding **which prime numbers** multiply together to make the original number

Use a factor tree



$54 = 2 \times 3 \times 3 \times 3$

ACT Math Strategy Tips

Below are several effective and timesaving strategies. These strategies are applicable to various topics on the ACT. They can also be used on all questions, regardless of difficulty level.

1. **Pick Numbers** – Look for math questions that have variables in the answer choices. You can usually substitute simple numbers in place of the variables and solve the question using numbers, not variables. Look for the answer choice that matches your answer when you substitute.

2. **Work Backwards** – This is a multiple-choice test. Therefore, one – and only one – answer choice must be correct. For math questions that ask for a numerical answer and have numbers as answer choices you can usually plug-in the answer choices into the problem until you find the one that works. This is especially useful on questions that require you to solve an equation.

3. **Use your Calculator** – NO MENTAL MATH! Avoid long calculations that your calculator can do for you simply. Familiarize yourself with the ACT Calculator Program Policy. Tedious calculations such as slope, distance, midpoint, and solving quadratics can be solved quickly, easily, and accurately using calculator programs. Ask your coach for information about how to put these programs on your calculator. You should use your calculator for even simple arithmetic problems. Use the calculator, get the correct answer and avoid silly errors. For the official ACT Calculator Policy visit: http://www.actstudent.org/faq/cas_functionality.html

4. **Each question is worth the same** - Therefore, students who struggle should spend the 60 minutes on easy/medium questions, getting them correct, and sacrificing some of the harder questions that they do not know how to do. Don't want to make silly mistakes because you are rushing to get to math questions you don't know how to do!

5. **Sketch it** – If a question describes a physical situation or a geometric shape but a diagram is not given, you should sketch one. Visuals are key to solving many geometry and trigonometry problems.

6. **Jump Around** – Generally speaking, math questions on the ACT go in order from easiest to hardest. However, once you get past question 30 or 40, it may be wise to skip around and pick questions that suit your strengths. Picking Numbers and Working Backwards questions are good candidates to pick.

7. **Don't be Afraid to Guess** – There is no guessing penalty on the ACT so there is no reason to ever leave a question blank. If you can eliminate incorrect answers you should, but NEVER LEAVE A QUESTION BLANK.

8. **Beware "Not enough information given"** – Sometimes the last answer to a math question will say something to the effect of "Not enough information given to determine the answer." Be very wary of this answer choice because it is most often not correct.

9. **Pace Yourself** – Questions on the math section go in order from easiest to hardest. You should allow yourself enough time to complete the more difficult questions as far as you can.

10. **Know Your Formulas** – Some questions give you the necessary formulas (Law of Sines & Law of Cosines for example) while others do not. You should know basic SOH-CAH-TOA formulas and Pythagorean Theorem at a bare minimum.

11. **Trust the Picture** – Unless a diagram specifically says “Not drawn to scale” you can safely assume the picture is drawn to scale. Use the diagram then make educated guesses and eliminate incorrect answer choices.

12. **Don't Stress the Triples** – On the math section, there will be three questions in a group that relate to a given diagram or situation. These are usually one easy, one medium, and one hard question. They are usually not related, so if you can't solve one you can still try the others.

13. **Guess A, D, or E on Hard Questions** – On the last part of the ACT Math (#45-60) choices A/F, D/J, and E/K are the most common. Don't guess B/G or C/H.

14. **Use Your Test Booklet** – Fill in or label the diagram with all information that the question gives. Write out a formula or equation that goes with the picture. Circle key words within the question.